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Design of reinforced concrete rectangular cross-sections according to the second generation of Eurocode

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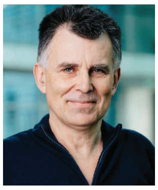

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Subject review

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Design of reinforced concrete rectangular cross-sections according to the second generation of Eurocode

This paper describes the design procedure for reinforced concrete rectangular cross-sections according to the standard FprEN 1992-1-1 (second generation of Eurocode 2). Design was carried out in two ways: using design tables for reinforced concrete rectangular sections and using a direct analytical procedure. The stress-strain diagram in the form of a parabola-rectangle was used for concrete, while for the reinforcing steel a bilinear stress-strain diagram with a horizontal post-elastic branch without strain limit was applied. For this reason, when designing the reinforced concrete cross-section, it is necessary to limit the strain in the compression zone of the concrete. Tables for the design of reinforced concrete rectangular sections were obtained using a new procedure in which the mechanical reinforcement ratio is varied. A direct analytical procedure was used for comparison, which can be used for the design of reinforced concrete rectangular sections without the need to use tables. The limit values of the coefficient of the height of the compression zone for pure bending were calculated according to the standard FprEN 1992-1-1.

Key words:

reinforced concrete cross-section, the second generation of Eurocode 2, bending, design, design tables, design charts

Pregledni rad

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Dimenzioniranje armiranobetonskih pravokutnih presjeka prema drugoj generaciji Eurokoda

U radu je opisan postupak dimenzioniranja armiranobetonskih pravokutnih presjeka prema normi FprEN 1992-1-1 (druga generacija Eurokoda 2). Dimenzioniranje je provedeno na dva načina: tablicama za dimenzioniranje armiranobetonskih pravokutnih presjeka i direktnim analitičkim postupkom. Korišten je proračunski dijagram naprezanje-deformacija betona u obliku parabole i pravca, dok je za čelik za armiranje korišten bilinearni proračunski dijagram naprezanje-deformacija s horizontalnom gornjom granom bez ograničenja deformacije. Zbog toga je kod dimenzioniranja armiranobetonskog presjeka potrebno ograničiti deformaciju u tlačnoj zoni betona. Tablice za dimenzioniranje armiranobetonskih pravokutnih presjeka dobivene su novim postupkom kod kojeg se varira mehanički koeficijent armiranja vlačnom armaturom. Za usporedbu primijenjen je direktni analitički postupak kojim se mogu dimenzionirati armiranobetonski pravokutni presjeci bez upotrebe tablica. Izračunane su limitirajuće vrijednosti koeficijenta visine tlačnog područja za čisto savijanje prema normi FprEN 1992-1-1.

Ključne riječi:

armiranobetonski presjek, druga generacija Eurokoda 2, savijanje, dimenzioniranje, tablice, dijagrami

1. Introduction

It is known that high-strength concrete is less ductile than normal-strength concrete, what results in change of the shape of the design stress-strain diagram for high-strength concrete. According to the currently valid standard HRN EN 1992-1-1 [1] (first generation of Eurocode 2), the design stress-strain diagrams for concrete differ depending on the concrete compressive strength class. The three following stress-strain diagrams can be used for the design of cross-sections subjected to bending for normal-strength concrete with characteristic strengths determined on cylinders from 12 to 60 MPa (compressive strength classes C12/15 to C50/60): in the form of a second degree parabola and a rectangle, a bilinear diagram or a diagram in the form of a rectangle. In all cases the ultimate strain is 3.5 ‰, while the strain at maximum strength is 2.0 ‰ for the parabola-rectangle diagram and 1.75 ‰ for the bilinear diagram.

For high-strength concrete - compressive strength classes C55/67, C60/75, C70/85, C80/95 and C90/105, as prescribed in the HRN EN 1992-1-1 standard, the ultimate strains in concrete are reduced compared to the strain values for normal-strength concrete, whereby a different value is assumed for each concrete class. On the other hand, strain at maximum compressive strength for high-strength concrete increases with the increase of the compressive strength class, with the curved part of the diagram (parabola) approaching the inclined line. In the case of rectangular stress distribution in the cross-section, both the factor that defines the effective height of the compression zone and the factor that defines the effective concrete strength depend on the concrete strength. For concrete classes C12/15 to C50/60, these effective factors are constant, while for concrete classes C55/67, C60/75, C70/85, C80/95 and C 90/105 they decrease depending on the class used. Due to these differences in stress-strain relationships for concrete, it was therefore necessary to create six different tables for the design of reinforced concrete cross-sections subjected to bending and/or interaction diagrams of cross-sections subjected to bending and axial force for each cross-section shape considered (rectangular, T-section, circular solid and hollow section, etc.). Each of the six mentioned aids (tables and interaction diagrams) had to be made regardless of the selected concrete stress-strain diagram: parabola-rectangle, bilinear or rectangular stress distribution at the effective height of the compression zone.

The use of stress-strain diagram for concrete in the form of parabola-rectangle is traditionally prevalent in Europe [2-6], while in the USA and in the UK the use of a uniform stress distribution at the effective height of the compression zone of the section (also reffered as rectangular stress distribution) is preferred [7, 8].

Aids for the design of reinforced concrete cross-sections in the form of tables and diagrams according to the currently valid first generation of Eurocode 2 are available in the literature (books, manuals and articles). For rectangular cross-sections: design tables can be found in [3-6]; interaction diagrams for members subjected to uniaxial bending and axial force can be found in [3, 5, 6] while for biaxial bending and axial force are available in [6]. Interaction diagrams for circular cross-section are available in [3, 5, 6, 9], while interaction diagrams for circular hollow sections for inner to outer diameter ratio of 0,9 can be found in [3]. According to the new standard FprEN 1992-1-1 [10] (second generation of Eurocode 2), the form of the stress-strain diagrams does not change depending on the concrete compressive strength class. The design stress-strain diagram is given in the form of a second degree parabola and a rectangle. Alternatively, a uniform distribution of compressive stresses over the effective height equal to 0,8 of the height of the compression zone can be used.

Due to this simplification, it is necessary to create only one table for the design of reinforced concrete rectangular cross-sections, which will be valid for all concrete classes, for the selected stress-strain design diagram (parabola-rectangle or uniform distribution).

According to the second generation of Eurocode 2 (as in the first generation of Eurocode 2), it is allowed to use a bilinear stress-strain relationship with a horizontal upper branch without strain limitation for reinforcing steel. If a bilinear relationship with an inclined upper branch is used, a strain limit must be introduced. In the USA [8] and according to the former British standard [11] and current British practice [7], only the stress-strain diagram with a horizontal upper branch without strain limitation for reinforcing steel is used. In the Republic of Croatia and in surrounding countries, there is a slightly different practice of limiting the strain in reinforcing steel, which can be traced back to the introduction of the limit states design, first as an alternative method to the allowable stress design method according to the Regulation on technical measures and conditions for concrete and reinforced concrete from 1971 [12], then as a mandatory method according to the Regulation on technical standards for concrete and reinforced concrete [13] and finally through the introduction of the European pre-standard ENV 1992-1-1 [14] and the current European standard HRN EN 1992-1-1 [1]. According to the regulations [12, 13] mentioned above, a bilinear stress-strain diagram with a horizontal upper branch and strain limit of 10 ‰ was used for all types of reinforcing steel prescribed at the time. At the same time (70s and 80s of the last century), a stress-strain diagram with a horizontal upper branch and strain limit of 5 ‰ (according to DIN 1045:1978) was used in Germany [15]. The CEB-FIB Model Code from 1978 [16], also recommended a bilinear stress-strain diagram for reinforcing steel with a horizontal upper branch and strain limit of 10 ‰. Some countries have introduced other strain limit values for reinforcing steel in their national standards, for example 7 ‰ in the Netherlands [17]. According to the pre-standard ENV 1992-1-1, the bilinear stress-strain diagram for reinforcing steel with inclined or horizontal upper branch may be used. When the inclined upper branch is used, the strain in the reinforcing steel is limited to 10

‰, while in case of application of the diagram with a horizontal upper branch, the standard states that the strain is not limited, although in some cases it may be appropriate to limit it (it is not specified to what value). In Croatia, the first design tables and interaction diagrams according to the European pre-standard ENV 1992-1-1 were created by Professor Ivan Tomičić [18]. He decided to use a bilinear stress-strain diagram for reinforcing steel with a horizontal upper branch with strain limit of 20 ‰. As stated in the Manual [18], the proposal of the German Committee for Reinforced Concrete from 1992 [19] was to use a bilinear diagram for reinforcing steel with a horizontal branch and strain limit of 20 ‰ during the temporary application of the pre-standard.

For the application of the currently valid generation of Eurocode 2 (HRN EN 1992-1-1 and other standards in the HRN EN 1992 series) in Croatia, design aids in the form of tables and/or interaction diagrams have been created [4, 5, 9]. The stress-strain diagram for reinforcing steel with a horizontal branch with maximum strain limited to 20 ‰ was applied. This is a consequence of the common practice from the time of application of the pre-standards (ENV 1992-1-1 and other standards of the EN 1992 series) when, due to the above-mentioned reason, the strain limit in the reinforcing steel of 20 ‰ was applied. According to the currently valid standard HRN EN 1992-1-1, bilinear stress-strain diagrams with inclined or horizontal upper branch can also be used. In case of inclined upper branch, it is necessary to check that the limit strain $\varepsilon_{ud} = 0.9\varepsilon_{uk}$ is not exceeded in design, whereby ε_{uk} is the characteristic strain in the reinforcing steel at maximum load. When using the diagram with horizontal upper branch, the strain limit control is not required. The authors of this paper have prepared tables for the design of rectangular cross-sections subjected to bending for an alternative application in teaching at the Faculty of Civil Engineering in Rijeka: parabola-rectangle stress-strain diagram for concrete and bilinear stress-strain diagram for reinforcing steel with horizontal upper branch without limiting the strain were used. The differences in the results regarding the required reinforcement area when using the design tables created with limitation of the reinforcing steel strain to 20 ‰ and without this limitation are negligible (≤ 1.0 %). These differences in the results (with or without strain limitation) would also be practically negligible for other strain limits (e.g. 5 ‰ or 10 ‰) if the stress-strain diagram for the reinforcing steel with horizontal upper branch is used, since for all the above strains the stress in the reinforcing steel is equal to the design yield strength.

In Germany, the national annex to the first part of the currently valid first generation of Eurocode 2 DIN EN 1992-1-1/NA:2013 [20] is applied. According to this national annex, the limiting strain in reinforcing steel when applying a bilinear stress-strain diagram with inclined upper branch is 25 ‰. For the application of the diagram with horizontal upper branch in [20], the need to limit the strains is not mentioned. However, design aids have been given in the German literature in which the strains in reinforcing steel were limited to 25 ‰, regardless of the application of the inclined or horizontal upper branch of the bilinear stress-strain diagram [2, 6]. This is due to the fact that this was prescribed in the German standard DIN 1045-1 [21], which preceded the first generation of Eurocode 2 in Germany.

According to the standard FprEN 1992-1-1 [10] (second generation of Eurocode 2), the design stress-strain diagrams for reinforcing steel have remained unchanged compared to the current Eurocode 2, i.e. the bilinear stress-strain diagram can be selected either with the inclined upper branch with strain limitation or with the horizontal upper branch without strain limitation.

Based on the above considerations, we have decided to use the stress-strain diagram for concrete in the form of a second degree parabola and a rectangle, while for the reinforcing steel we have used a bilinear stress-strain diagram with a horizontal upper branch without strain limitation. If we use the stress-strain diagram for the reinforcing steel with a horizontal upper branch without limiting strains, the limit strain in the reinforcing steel is never reached as it is at infinity. The consequence of the above assumptions is that we have to limit the strains in the compression zone of the concrete to -3.5 ‰, when we design the reinforced concrete section.

In the past, reinforced concrete cross-sections were designed according to the method of allowable stresses (classical theory) - in the Republic of Croatia for the last time as an alternative method according to the Regulation on technical standards for concrete and reinforced concrete [13]. The design of reinforced concrete cross-sections according to the allowable stress method is based on the verification that the highest stresses in concrete and reinforcement, that may occur during construction and use, do not exceed the allowable stresses. This method is based on the basic assumptions of a cracked cross-section. Due to numerous shortcomings, the allowable stress design is no longer used [2].

In the preparation of tables for the design of reinforced concrete rectangular cross-sections, a new procedure was used in which the mechanical reinforcement ratio is varied, since the required area of tensile reinforcement depends linearly on the mechanical ratio. The tables for the design of reinforced concrete rectangular sections were created using the computer program Mathcad 15 [22].

In addition to the tables for the design of reinforced concrete rectangular cross-sections, a direct analytical procedure has been proposed in this paper, which can be used for the design of reinforced concrete rectangular cross-sections without the need to use tables (the problem is reduced to the solution of a quadratic equation).

In the standard FprEN 1992-1-1 [10] (second generation of Eurocode 2) no limit values have been defined for the coefficient of the height of the compression zone for pure bending, which serves as a boundary between singly and doubly reinforced sections. For this reason, limit values for the coefficient of the height of the compression zone have been proposed, which can be used for the design of reinforced concrete cross-sections according to the FprEN 1992-1-1 standard.

2. Stress-strain design diagrams

Compressive stress and strain values in concrete and reinforcing steel in this paper assume a negative sign, while tensile stresses and strains assume a positive sign. Strains in concrete and reinforcing steel are expressed in ‰.

2.1. Design diagram for concrete

According to FprEN 1992-1-1 [10] (second generation of Eurocode 2) the design value of concrete compressive strength is taken as:

$$f_{cd} = \eta_{cc} \cdot k_{tc} \cdot \frac{f_{ck}}{\gamma_c} \quad (1)$$

where f_{ck} is the characteristic concrete compressive strength, γ_C is the partial factor for concrete (γ_C = 1.50 for persistent and transient design situations), η_{cc} is a factor to account for the difference between the undisturbed compressive strength of a cylinder and the effective compressive strength that can be developed in the structure, while k_{tc} is a factor that accounts for the effect of high sustained loads and time of loading on the concrete compressive strength.

$$\eta_{cc} = \left(\frac{f_{ck,ref}}{f_{ck}}\right)^{1/3} \leq 1.0 \quad (2)$$

In FprEN 1992-1-1 [10] (second generation of Eurocode 2) the following recommended values are provided:
- $f_{ck,ref}$ = 40 MPa
- k_{tc} = 1.00 for t_{ref} ≤ 28 days for concretes with classes CR (rapid strength development) and CN (normal strength development) and t_{ref} ≤ 56 days for concretes with class CS (slow strength development) where the design loading is not expected for at least three months after casting, with t_{ref} as the age of concrete at which the concrete strength is determined.
- k_{tc} = 0.85 for other cases including when f_{ck} is replaced by $f_{ck}(t)$.

In the National annex, according to the standard FprEN 1992-1-1 [10] (second generation of Eurocode 2), other values may be given (Croatian National Annex in still in the development phase).

A simplified concrete stress-strain relationship (parabola -rectangle), shown in Figure 1, can be used for design of cross-sections subjected to bending with an axial force. Stresses in concrete can be expressed in the following terms:

$$\sigma_{cd} = -f_{cd}\left[1-\left(1+\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^2\right] \quad \text{for } -\varepsilon_{c2} \leq \varepsilon_c \leq 0 \quad (3)$$

$$\sigma_{cd} = -f_{cd} \quad \text{for } -\varepsilon_{cu} \leq \varepsilon_c \leq -\varepsilon_{c2} \quad (4)$$

$$\sigma_{cd} = 0 \text{ MPa} \quad \text{for } \varepsilon_c > 0 \quad (5)$$

where ε_{c2} = 2 ‰ while ε_{cu} = 3.5 ‰.

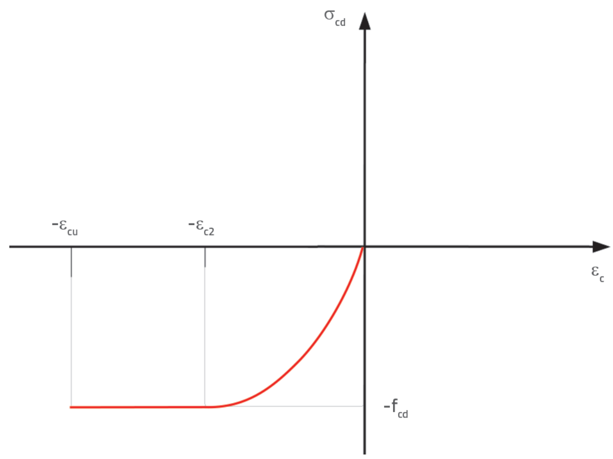

Figure 1. Stress - strain diagram for concrete

2.2. Design diagram for reinforcing steel

According to FprEN 1992-1-1 [10] (second generation of Eurocode 2) the bilinear stress-strain diagram for reinforcing steel with horizontal upper (post-elastic) branch without strain limitation (Figure 2) can be used for the design of cross-sections subjected to bending. The symbols in Figure 2 are as follows: f_{yd} is the design yield strength of reinforcing steel, f_{yk} is the characteristic yield strength of reinforcing steel, while γ_S is the partial factor for reinforcing steel (γ_S = 1.15 for persistent and transient design situations).

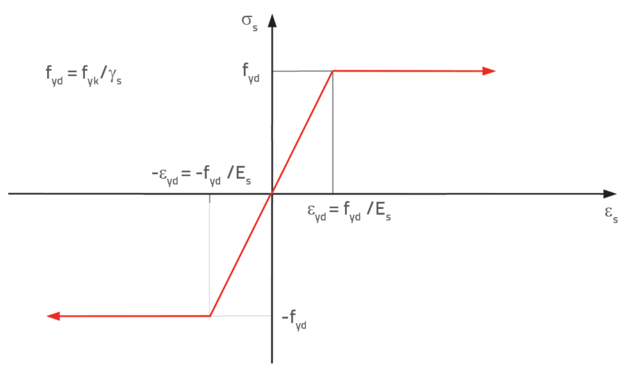

Figure 2. Stress – strain diagram for reinforcing steel

The stress in reinforcement (according to Figure 2) is described by the following expressions:

$$\sigma_s = -f_{yd} \quad \text{for } \varepsilon_s \leq -\varepsilon_{yd} \quad (6)$$

$$\sigma_s = \varepsilon_s \cdot E_s \quad \text{for } -\varepsilon_{yd} < \varepsilon_s < \varepsilon_{yd} \quad (7)$$

$$\sigma_s = f_{yd} \quad \text{for } \varepsilon_s \geq \varepsilon_{yd} \quad (8)$$

where E_s is the modulus of elasticity of steel (E_s = 200 GPa), while ε_{yd} is the design yield strain in the reinforcement.

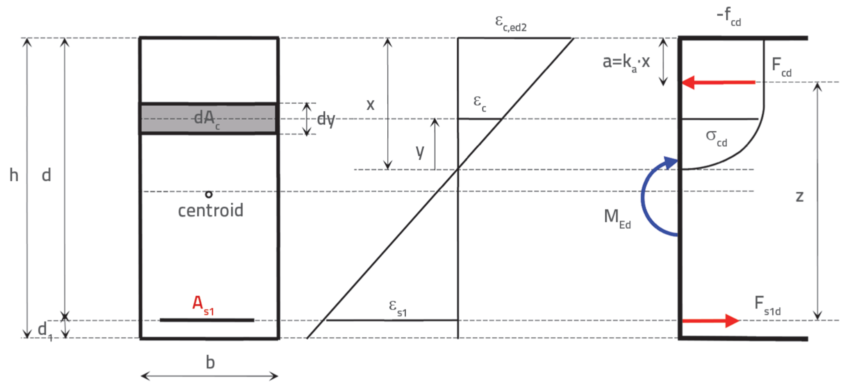

Figure 3. Rectangular cross-section with distribution of strains, stresses and forces

Level arm of internal forces equals to (Figure 3):

$$z = d-k_a \cdot x = d-k_a \cdot (\xi \cdot d) = (1-k_a \cdot \xi) \cdot d = \zeta \cdot d \quad (12)$$

with:

$$\zeta = 1-k_a \cdot \xi \quad (13)$$

presenting the coefficient of the level arm of internal forces, while k_a is the position coefficient of the resultant of the design compressive force in concrete. The differential area of concrete is obtained according to Figure 3:

$$dA_c = b \cdot dy \quad (14)$$

3. Tables for design of reinforced concrete rectangular cross-sections

3.1. Cross-sectional description

The description of the rectangular cross-section with reinforcement is presented in Figure 3. It is assumed that plane cross-sections remain plane even after deformation (Bernoulli hypothesis of plane sections), which has the consequence that all points of the deformed section will be situated in the same plane (plane of strain) which passes through the neutral axis x of the cross-section. The following assumptions are used in design:

- plain sections remain plane,
- strains in bonded reinforcement (both in tension and in compression) are the same as the strain in the surrounding concrete,
- the tensile strength of concrete is ignored,
- the stresses in the concrete in compression are derived from the design stress-strain relationship given in 2.1
- the stresses in the reinforcing steel are derived from the design stress-strain relationship given in 2.2.

Based on the similarity of the triangles (Figure 3) we obtain:

$$\frac{|\varepsilon_{c,ed2}|}{x} = \frac{|\varepsilon_{c,ed2}| + \varepsilon_{s1}}{d} \quad (9)$$

where $\varepsilon_{c,ed2}$ is the strain in the concrete at the top edge, ε_{s1} is the strain in the tensile reinforcement, *x* is the depth of the neutral axis while *d* is the distance between the center of gravity of tensile reinforcement and the top edge of the concrete cross-section (effective depth of the cross-section).
Expression (9) can be presented as:

$$x = \frac{|\varepsilon_{c,ed2}|}{|\varepsilon_{c,ed2}| + \varepsilon_{s1}} d = \xi \cdot d \quad (10)$$

where:

$$\xi = \frac{|\varepsilon_{c,ed2}|}{|\varepsilon_{c,ed2}| + \varepsilon_{s1}} \quad (11)$$

is the coefficient of the height of the compression zone.

3.2. Coefficients α_v and k_a

The design compressive force in concrete is defined according to the following expression:

$$F_{cd} = \left| \int_{A_c} \sigma_{cd} \cdot (dA_c) \right| \quad (15)$$

When expression (14) is inserted into expression (15) we obtain:

$$F_{cd} = \left| \int_0^x \sigma_{cd} \cdot (b \cdot dy) \right| = \left| b \cdot \int_0^x \sigma_{cd} \cdot dy \right| \quad (16)$$

Based on the similarity of triangles (Figure 3), we obtain

$$\frac{\varepsilon_c}{y} = \frac{\varepsilon_{c,ed2}}{x} \quad (17)$$

Expression (17) can be presented as:

$$y = \frac{x}{\varepsilon_{c,ed2}} \varepsilon_c \quad (18)$$

By differentiating expression (18), we get:

$$dy = \frac{x}{\varepsilon_{c,ed2}} d\varepsilon_c \quad (19)$$

When expression (19) is inserted into expression (16) we obtain:

$$F_{cd} = \left| b \cdot \int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot \left(\frac{x}{\varepsilon_{c,ed2}} d\varepsilon_c \right) \right| = \left| b \cdot x \cdot \left(\frac{1}{\varepsilon_{c,ed2}} \int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot d\varepsilon_c \right) \right| \quad (20)$$

If we multiply and divide expression (20) by f_{cd}, we get:

$$F_{cd} = b \cdot x \cdot f_{cd} \left| \frac{1}{\varepsilon_{c,ed2} \cdot f_{cd}} \int_0^{\varepsilon_{c,ed2}} \sigma_c \cdot d\varepsilon_c \right| = b \cdot x \cdot f_{cd} \cdot \alpha_v \quad (21)$$

where α_v is the coefficient of fullness.

$$\alpha_v = \left| \frac{1}{\varepsilon_{c,ed2} \cdot f_{cd}} \int_0^{\varepsilon_{c,ed2}} \sigma_c \cdot d\varepsilon_c \right| \quad (22)$$

Once expressions (3) and (4) are inserted into expression (22) and after integration, we obtain:

$$\alpha_v = \frac{\left|\varepsilon_{c,ed2} \cdot \left(\varepsilon_{c,ed2} + 6\right)\right|}{12} \quad \text{for} \quad -\varepsilon_{c2} \leq \varepsilon_{c,ed2} < 0 \quad (23)$$

$$\alpha_v = \frac{\left|3 \cdot \varepsilon_{c,ed2} + 2\right|}{3 \cdot \left|\varepsilon_{c,ed2}\right|} \quad \text{for} \quad \varepsilon_{cu} \leq \varepsilon_{c,ed2} < -\varepsilon_{c2} \quad (24)$$

When we calculate the sum of the static moments of the design compressive force in concrete with respect to the upper edge of the cross-section, we get:

$$F_{cd} \cdot a = \int_{A_c} \sigma_{cd} \cdot (x - y) \cdot dA_c \quad (25)$$

Inserting expression (15) into expression (25) yields

$$\left(\int_{A_c} \sigma_{cd} \cdot dA_c \right) \cdot a = \int_{A_c} \sigma_{cd} \cdot (x - y) \cdot dA_c \quad (26)$$

while dividing expression (26) by amounts to $\int_{Ac} \sigma_{cd} \cdot dA_c$

$$a = \frac{\int_{A_c} \sigma_{cd} \cdot (x - y) \cdot dA_c}{\int_{A_c} \sigma_{cd} \cdot dA_c} = \frac{\int_{A_c} \sigma_{cd} \cdot x \cdot dA_c}{\int_{A_c} \sigma_{cd} \cdot dA_c} - \frac{\int_{A_c} \sigma_{cd} \cdot y \cdot dA_c}{\int_{A_c} \sigma_{cd} \cdot dA_c} \quad (27)$$

that is

$$a = x - \frac{\int_{A_c} \sigma_{cd} \cdot y \cdot dA_c}{\int_{A_c} \sigma_{cd} \cdot dA_c} \quad (28)$$

When expression (14) is introduced into expression (28) we obtain:

$$a = x - \frac{\int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot y \cdot (b \cdot dy)}{\int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot (b \cdot dy)} = x - \frac{\int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot y \cdot dy}{\int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot dy} \quad (29)$$

Inserting expressions (18) and (19) into expression (29) ammounts to:

$$a = x - \frac{\int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot \left(\frac{x}{\varepsilon_{c,ed2}} \cdot \varepsilon_c \right) \cdot \left(\frac{x}{\varepsilon_{c,ed2}} \cdot d\varepsilon_c \right)}{\int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot \left(\frac{x}{\varepsilon_{c,ed2}} \cdot d\varepsilon_c \right)} = x - \frac{\frac{x}{\varepsilon_{c,ed2}} \int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot \varepsilon_c \cdot d\varepsilon_c}{\int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot d\varepsilon_c} \quad (30)$$

Dividing expression (30) by x gives:

$$\frac{a}{x} = 1 - \frac{\frac{1}{\varepsilon_{c,ed2}} \int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot \varepsilon_c \cdot d\varepsilon_c}{\int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot d\varepsilon_c} \quad (31)$$

Left side of expression (31) represents the position coefficient of the resultant of the design compressive force in concrete:

$$k_a = 1 - \frac{\frac{1}{\varepsilon_{c,ed2}} \int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot \varepsilon_c \cdot d\varepsilon_c}{\int_0^{\varepsilon_{c,ed2}} \sigma_{cd} \cdot d\varepsilon_c} \quad (32)$$

When expressions (3) and (4) are inserted into expression (32) and after integration, we obtain the following expressions:

$$k_a = \frac{\left|\varepsilon_{c,ed2} + 8\right|}{4 \cdot \left|\varepsilon_{c,ed2} + 6\right|} \quad \text{for} \quad -\varepsilon_{c2} \leq \varepsilon_{c,ed2} < 0 \quad (33)$$

$$k_a = \frac{\left|3 \cdot \varepsilon_{c,ed2}^2 + 4 \cdot \varepsilon_{c,ed2} + 2\right|}{2 \cdot \left|3 \cdot \varepsilon_{c,ed2} + 2\right| \cdot \left|\varepsilon_{c,ed2}\right|} \quad \text{for} \quad -\varepsilon_{cu} \leq \varepsilon_{c,ed2} < -\varepsilon_{c2} \quad (34)$$

Here, we always limit the strain in concrete at the upper edge $\varepsilon_{c,ed2}$ to $-\varepsilon_{cu}$ = -3.5 ‰ because we do not limit the strain in the reinforcing steel. From expression (24) we obtain that the coefficient of fullness is ε_v(-3.5 ‰) = 0.810, while from expression (34) the position coefficient of the resultant of the design compressive force in concrete equals to k_a(-3.5 ‰) = 0.416.

3.3. Equilibrium equations

The design force in the tensile reinforcement is obtained from:

$$F_{s1d} = \sigma_{s1d} \cdot A_{s1} \quad (35)$$

where σ_{s1d} is the design stress in tensile reinforcement, while A_{s1} is the area of tensile reinforcement.

The load bearing moment by which the cross-section withstands the bending action, calculated with respect to the center of gravity of the tensile reinforcement, amounts to:

$$M_{Rd} = F_{cd} \cdot z \quad (36)$$

The load bearing moment by which the cross-section withstands the bending action, calculated with respect to the center of gravity of the compressive force in concrete, amounts to:

$$M_{Rd} = F_{s1d} \cdot z \quad (37)$$

In cross-sections subjected to bending moment, it is necessary to satisfy the condition:

$$M_{Ed} \leq M_{Rd} \quad (38)$$

where M_{Ed} is the design acting bending moment, while M_{Rd} is the moment of resistance of the cross-section.

The equilibrium equation is used for design:

$$M_{Ed} = M_{Rd} = F_{cd} \cdot z \quad (39)$$

or

$$M_{Ed} = M_{Rd} = F_{s1d} \cdot z \quad (40)$$

Based on the equilibrium equation (39) and expressions (21), (10) and (12), we obtain:

$$M_{Ed} = F_{cd} \cdot z = (b \cdot x \cdot f_{cd} \cdot \alpha_v)(\zeta \cdot d) = (b \cdot (\xi \cdot d) \cdot f_{cd} \cdot \alpha_v)(\zeta \cdot d) \quad (41)$$

i.e.:

$$M_{Ed} = (\alpha_v \cdot \xi \cdot \zeta)(b \cdot d^2 \cdot f_{cd}) \quad (42)$$

Dividing expression (42) by $b \cdot d^2 \cdot f_{cd}$ gives

$$\mu_{Ed} = \frac{M_{Ed}}{b \cdot d^2 \cdot f_{cd}} = \mu_{Rd} = \alpha_v \cdot \xi \cdot \zeta \quad (43)$$

where μ_{Ed} is the dimensionless design bending moment, while μ_{Rd} is the dimensionless moment of resistance.
The required reinforcement area can be determined from expression (40), by applying expressions (35) and (12):

$$M_{Ed} = F_{s1d} \cdot z = (\sigma_{s1} \cdot A_{s1})(\zeta \cdot d) \quad (44)$$

If we assume that the tensile reinforcement yielded, this means that the stress in the tensile reinforcement is equal to the design yield strength $\sigma_{s1} = f_{yd}$ (Figure 2), then expression (44) becomes:

$$M_{Ed} = (f_{yd} \cdot A_{s1})(\zeta \cdot d) \quad (45)$$

Dividing expression (45) by $f_{yd} \cdot \zeta \cdot d$ we obtain:

$$A_{s1} = \frac{M_{Ed}}{\zeta \cdot d \cdot f_{yd}} \quad (46)$$

Alternatively, the required area of tensile reinforcement can be determined from the sum of horizontal forces in the cross-section:

$$\sum H = 0 \quad (47)$$

$$F_{s1d} - F_{cd} = 0 \quad (48)$$

$$F_{s1d} = F_{cd} \quad (49)$$

If expressions (21), (35) and (10) are inserted into the previous expression (49) and if we assume that the tensile reinforcement yielded ($\sigma_{s1} = f_{yd}$), we obtain:

$$F_{yd} \cdot A_{s1} = b \cdot (\xi \cdot d) \cdot f_{cd} \cdot \alpha_v \quad (50)$$

Dividing expression (50) by f_{yd} gives the required area of reinforcement:

$$A_{s1} = \alpha_v \cdot \xi \cdot \frac{f_{cd}}{f_{yd}} b \cdot d = \omega_1 \frac{f_{cd}}{f_{yd}} b \cdot d = \rho_1 \cdot b \cdot d \quad (51)$$

where:

$$\omega_1 = \alpha_v \cdot \xi \quad (52)$$

is the mechanical reinforcement ratio, while

$$\rho_1 = \omega_1 \frac{f_{cd}}{f_{yd}} \quad (53)$$

is the reinforcement ratio.

3.4. Limit values for pure bending

Limit values for pure bending are required to distinguish between singly and doubly reinforced cross-sections. The standard FprEN 1992-1-1 [10] (second generation of Eurocode 2) has not defined limit values for pure bending, therefore limit values for pure bending are derived and proposed here.
The limit value of the coefficient of the height of the compression zone $\xi_{lim} = x / d$ is obtained by setting the ratio of the bending moment after redistribution (linear analysis with redistribution) to the elastic bending moment ξ_M to 1.
The standard FprEN 1992-1-1 allows the use of linear analysis with redistribution, without checking the rotational capacity, if:

$$\delta_1 \geq \frac{1}{1 + 0.7 \frac{\varepsilon_{cu}}{1000} \frac{E_s}{f_{yd}}} + \frac{x}{d} \quad (54)$$

Structures are still predominantly designed according to the linear theory, which would mean that $\delta_M = 1$ (there is no redistribution of the bending moment). If we equate expression (54) with 1, we obtain the following expression:

$$\frac{1}{1 + 0.7 \frac{\varepsilon_{cu}}{1000} \frac{E_s}{f_{yd}}} + \frac{x}{d} = 1 \quad (55)$$

i.e. the limiting value of the coefficient of the height of the pressure area ξ_{lim} is obtained:

$$\xi_{lim} = \frac{x}{d} = 1 - \frac{1}{1 + 0.7 \frac{\varepsilon_{cu}}{1000} \frac{E_s}{f_{yd}}} \quad (56)$$

Based on the similarity of the triangles (Figure 3) and expression (9) and with $\varepsilon_{c,ed2} = -\varepsilon_{cu} = -3{,}5$ ‰ the limit value of the tensile strain in the reinforcement is obtained:

$$\varepsilon_{s1,lim} = \frac{|-3.5\text{ ‰}|(1 - \xi_{lim})}{\xi_{lim}} \quad (57)$$

Table 1. Limit values for the case of pure bending (E_s = 200 GPa, γ_s = 1.15)

f_{yk} [MPa]	f_{yd} [MPa]	$\varepsilon_{c.ed2.lim}$ [‰]	$\varepsilon_{s1.lim}$ [‰]	ξ_{lim}	ζ_{lim}	$\mu_{Rd.lim}$	$\omega_{1.lim}$
400	347.826	-3.5	2.484	0.585	0.757	0.358	0.473
450	391.304	-3.5	2.795	0.556	0.769	0.346	0.450
500	434.783	-3.5	3.106	0.530	0.780	0.334	0.429
550	478.261	-3.5	3.416	0.506	0.789	0.323	0.410
600	521.739	-3.5	3.727	0.484	0.799	0.313	0.392
700	608.696	-3.5	4.348	0.446	0.814	0.294	0.361

For the design of concrete structures according to the FprEN 1992-1-1 standard, reinforcing steel with the following characteristic yield strengths f_{yk} can be used: 400 MPa, 450 MPa, 500 MPa, 550 MPa, 600 MPa and 700 MPa. Therefore, the limit values for pure bending must be determined for six different strength classes (yield strengths) of the reinforcing steel.
These limit values for pure bending for different characteristic yield strengths f_{yk} are given in Table 1.

3.5. Design tables

The possible range of strain distribution in a reinforced concrete cross-section of general shape for the case of pure bending is presented in Figure 4.
A reinforced concrete cross-section will be in the ultimate limit state when at least one strain limit is reached (Figure 4): either in the concrete ($-\varepsilon_{cu}$) or in the reinforcing steel (ε_{ud}). If we use the design stress-strain diagram for the reinforcing steel with a horizontal upper branch without strain limit (as in Figure 2), the ultimate strain in the reinforcing steel will never be reached. Therefore, for the reinforced concrete cross-section to be in the ultimate limit state, the strain in the compression zone of the concrete must be reached and limited to $\varepsilon_{c,ed2} = -\varepsilon_{cu}$ = -3.5 ‰.
Tables for the design of reinforced concrete rectangular cross-sections could be obtained so that the strain in the compression zone of the concrete is limited to $\varepsilon_{c,ed2} = -\varepsilon_{cu}$ = -3.5 ‰, while the strain in the tensile reinforcement is varied (rotation around point A in Figure 4).
This paper presents a new procedure for creating table for the design of reinforced concrete rectangular cross-sections in such a way that the mechanical reinforcement ratio ω_1 is varied, because from expression (51) the required area of tensile reinforcement is linearly dependent on the mechanical reinforcement ratio ω_1.
In this way, table for design of reinforced concrete rectangular sections is obtained where the maximum absolute error of the required area of tensile reinforcement is equal, which corresponds to the varying step of the mechanical reinforcement ratio ω_1 (0,010).
When creating table for design of reinforced concrete rectangular cross-sections, by rotation around point A, we know strain $\varepsilon_{c,ed2} = -\varepsilon_{cu}$ = -3.5 ‰, ω_1 varies from 0.010 to 0.540 with a step of 0.010, while the unknown is ε_{s1}. If expressions (11) and (24) are inserted into (52) with $\varepsilon_{c,ed2} = -\varepsilon_{cu}$ = -3.5 ‰, we obtain the tensile strain in the reinforcement ε_{s1} as a function of ω_1:

$$\varepsilon_{s1} = \left|-3.5\ ‰\right| \left(\frac{\alpha_v(-3.5\ ‰)}{\omega_1} - 1 \right) \quad (58)$$

The tensile strain in the reinforcement ε_{s1} as a function of ω_1 can be determined directly from expression (58) only because the strain in the compression zone of the concrete is $\varepsilon_{c,ed2} = -\varepsilon_{cu}$ = -3.5 ‰, and consequently the coefficient of fullness α_v(-3.5 ‰) = 0.810 is also a constant. If the strain in the compression zone of the concrete $\varepsilon_{c,ed2}$ ranged from 0 ‰ to -3.5 ‰, then the tensile strain in the reinforcement ε_{s1} would have to be determined by solving a non-linear equation with one unknown for each row of the table for the design of reinforced concrete rectangular cross-sections.

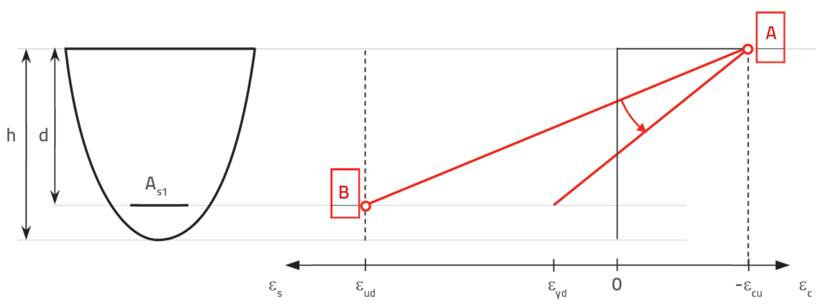

Figure 4. Possible strain distributions in a reinforced concrete cross-section of general shape for the case of pure bending

Table for the design of reinforced concrete rectangular cross-sections is obtained such that $\varepsilon_{c,ed2} = -\varepsilon_{cu}$ = -3.5 ‰, ω_1 varies from 0.010 to 0.540 with a step of 0.010, while the tensile strain in the reinforcement ε_{s1} is obtained as a function of ω_1 using expression (58). The other values in the table ξ, ζ and μ_{Ed} result from expressions (11), (13) and (43).
The table for the design of reinforced concrete rectangular cross-sections

was created using the computer program Mathcad 15 [22] and is shown in Table 2.

Table 2. Table for design of reinforced concrete rectangular cross-sections, α_v(-3.5 ‰) = 0.810 and k_a(-3.5 ‰) = 0.416

$\xi = x/d$	$\zeta = z/d$	μ_{Ed}	ω_1
0.012	0.995	0.010	0.010
0.025	0.990	0.020	0.020
0.037	0.985	0.030	0.030
0.049	0.979	0.039	0.040
0.062	0.974	0.049	0.050
0.074	0.969	0.058	0.060
0.086	0.964	0.067	0.070
0.099	0.959	0.077	0.080
0.111	0.954	0.086	0.090
0.124	0.949	0.095	0.100
0.136	0.943	0.104	0.110
0.148	0.938	0.113	0.120
0.161	0.933	0.121	0.130
0.173	0.928	0.130	0.140
0.185	0.923	0.138	0.150
0.198	0.918	0.147	0.160
0.210	0.913	0.155	0.170
0.222	0.908	0.163	0.180
0.235	0.902	0.171	0.190
0.247	0.897	0.179	0.200
0.259	0.892	0.187	0.210
0.272	0.887	0.195	0.220
0.284	0.882	0.203	0.230
0.296	0.877	0.210	0.240
0.309	0.872	0.218	0.250
0.321	0.866	0.225	0.260
0.334	0.861	0.233	0.270
0.346	0.856	0.240	0.280
0.358	0.851	0.247	0.290
0.371	0.846	0.254	0.300
0.383	0.841	0.261	0.310
0.395	0.836	0.267	0.320
0.408	0.830	0.274	0.330
0.420	0.825	0.281	0.340
0.432	0.820	0.287	0.350
0.445	0.815	0.293	0.360
0.457	0.810	0.300	0.370
0.469	0.805	0.306	0.380
0.482	0.800	0.312	0.390

Table 2. Table for design of reinforced concrete rectangular cross-sections, α_v(-3.5 ‰) = 0.810 and k_a(-3.5 ‰) = 0.416 - continuation

$\xi = x/d$	$\zeta = z/d$	μ_{Ed}	ω_1
0.494	0.794	0.318	0.400
0.506	0.789	0.324	0.410
0.519	0.784	0.329	0.420
0.531	0.779	0.335	0.430
0.544	0.774	0.341	0.440
0.556	0.769	0.346	0.450
0.568	0.764	0.351	0.460
0.581	0.758	0.356	0.470
0.593	0.753	0.362	0.480
0.605	0.748	0.367	0.490
0.618	0.743	0.372	0.500
0.630	0.738	0.376	0.510
0.642	0.733	0.381	0.520
0.655	0.728	0.386	0.530
0.667	0.723	0.390	0.540

Tables for the design of reinforced concrete rectangular cross-sections (Table 2) can also be displayed graphically as a diagram of the coefficient of the level arm of internal forces ζ versus the dimensionless design bending moment μ_{Ed}, or as well as a diagram of the mechanical reinforcement ratio ω_1 versus the dimensionless design bending moment μ_{Ed} (Figure 5). The diagram of the coefficient of the level arm of internal forces ζ as a function of the dimensionless design bending moment μ_{Ed} can be used as a substitute for the design tables. The diagram of the mechanical reinforcement ratio ω_1 as a function of the dimensionless bending moment μ_{Ed} can be used to determine the moment of resistance M_{Rd}.

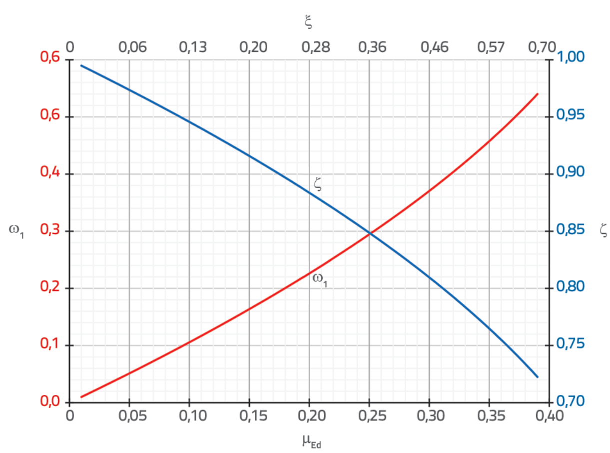

Figure 5. Design chart: functions $\zeta(\mu_{Ed})$ and $\omega_1(\mu_{Ed})$

4. Direct analytical procedure

With the direct analytical procedure, it is possible to design reinforced concrete rectangular cross-sections according to the standard FprEN 1992-1-1 (second generation of Eurocode 2) without using the tables for the design given in Table 2 [3]. In the direct analytical procedure, the depth of the neutral axis x must be determined by solving the quadratic equation, whereupon the required area of the tensile reinforcement A_{s1} is obtained. The expressions for the direct analytical procedure can only be obtained if the strain in the compression zone of the concrete is limited to $\varepsilon_{c,ed2} = -\varepsilon_{cu} = -3.5$ ‰, as shown in Chapter 3.

The description of the rectangular cross-section with reinforcement is shown in Figure 3, which we used when creating tables for design of the reinforced concrete rectangular cross-sections. If the expressions (21) and (12) are inserted into expression (39), we obtain:

$$M_{Ed} = M_{Rd} = F_{cd} \cdot z = (\alpha_v(-3{,}5\text{ ‰}) \cdot b \cdot x \cdot f_{cd}) \cdot (d - k_a(-3{,}5\text{ ‰}) \cdot x) \qquad (59)$$

The strain in the compression zone of concrete is $\varepsilon_{c,ed2} = -\varepsilon_{cu} = -3.5$ ‰,, coefficient of fullness $\alpha_v(-3.5\text{ ‰}) = 0.810$, position coefficient of the resultant of the design compressive force in concrete $k_a(-3.5\text{ ‰}) = 0.416$, and the only unknown in expression (59) is the depth of the neutral axis x.

By arranging the expression (59), the quadratic equation is obtained:

$$\frac{k_a(-3.5\text{‰})}{d} x^2 - x + \frac{M_{Ed}}{\alpha_v(-3.5\text{‰}) \cdot b \cdot d \cdot f_{cd}} = 0 \qquad (60)$$

which has two solutions:

$$x_{1,2} = \frac{d}{2 \cdot k_a(-3.5\text{‰})} \left[1 \pm \sqrt{1 - \frac{4 \cdot k_a(-3.5\text{‰}) \cdot M_{Ed}}{\alpha_v(-3.5\text{‰}) \cdot b \cdot d^2 \cdot f_{cd}}} \right] \qquad (61)$$

The first solution x_1 from (61) has no physical meaning because we obtain $x_1 > d$, therefore the second solution x_2 gives the depth of the neutral axis $x = x_2$:

$$x = \frac{d}{2 \cdot k_a(-3.5\text{‰})} \left[1 - \sqrt{1 - \frac{4 \cdot k_a(-3.5\text{‰}) \cdot M_{Ed}}{\alpha_v(-3.5\text{‰}) \cdot b \cdot d^2 \cdot f_{cd}}} \right] \qquad (62)$$

The tensile strain in the reinforcement ε_{s1} is obtained using expression (9), substituting the strain in the compression zone of the concrete with $\varepsilon_{c,ed2} = -\varepsilon_{cu} = -3.5$ ‰:

$$\varepsilon_{s1} = \frac{|-3.5\text{‰}|(d - x)}{x} \qquad (63)$$

If expressions (35) and (21) are inserted into expression (48), and assuming that the tensile reinforcement yielded ($\sigma_{s1d} = f_{yd}$) and that strain in the compression zone of concrete is $\varepsilon_{c,ed2} = -\varepsilon_{cu} = -3.5$ ‰ we obtain:

$$F_{s1d} - F_{cd} = (f_{yd} \cdot A_{s1}) - (\alpha_v(-3.5\text{ ‰}) \cdot b \cdot x \cdot f_{cd}) = 0 \qquad (64)$$

By arranging the expression (64), the required area of tensile reinforcement amounts to:

$$A_{s1} = \frac{\alpha_v(-3.5\text{‰}) \cdot b \cdot x \cdot f_{cd}}{f_{yd}} \qquad (65)$$

The limiting bending moment for a singly reinforced cross-section is obtained via (43):

$$M_{R,d,lim} = \mu_{R,d,lim} \cdot b \cdot d^2 \cdot f_{cd} \qquad (66)$$

while the limiting dimensionless moment of resistance $\mu_{Rd,lim}$ is determined using Table 1.

The obtained expressions are generally valid regardless of the selected concrete stress-strain diagram. If we use the stress-strain diagram of concrete in the form of a rectangle (uniform distribution), which is allowed according to the standard FprEN 1992-1-1, we would use the coefficient of fullness $\alpha_v(-3.5\text{ ‰}) = 0.8$, and the coefficient of the position of the resultant design compressive forces in concrete $k_a(-3.5\text{ ‰}) = 0.4$.

5. Numerical examples

5.1. Example 1

Calculate the required reinforcement area for a rectangular cross-section b = 30 cm and h = 65 cm (d = 61 cm) (see Figure 3), which is subjected to bending moment from permanent action M_G = 40 kNm and bending moment from variable action M_Q = 65 kNm. The partial factor for the permanent action is γ_G = 1.35, while the partial factor for the variable action is γ_Q = 1.50. Materials: concrete C25/30 and reinforcing steel B500. The age of concrete at which the compressive strength is determined is t_{ref} = 28 days. Cement type CN (normal strength development) is used for concrete. The minimum and maximum reinforcement will not be checked in the example.

For comparison purposes, it is necessary to design the reinforced concrete rectangular cross-section using both tables and the direct analytical procedure.

Factor η_{cc} – expression (2):

$$\eta_{cc} = \left(\frac{f_{ck,ref}}{f_{ck}} \right)^{\frac{1}{3}} \le 1.0$$

$$\eta_{cc} = \left(\frac{40}{25} \right)^{\frac{1}{3}} = 1.17 \le 1.0$$

$$\eta_{cc} = 1.0$$

Factor k_{tc} for the age of concrete at which the compressive strength is determined t_{ref}=28 days and cement class CN (normal strength development) equals to

$$k_{tc} = 1.0$$

Design concrete compressive strength - expression (1):

$$f_{cd} = \eta_{cc} \cdot k_{tc} \cdot \frac{f_{ck}}{\gamma_C} = 1.0 \cdot 1.0 \cdot \frac{25}{1.5} = 16.67 \text{ MPa}$$

Design yield strength of reinforcing steel (Figure 2):

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{500}{1.15} = 434.78 \text{ MPa}$$

Design bending moment:

$M_{Ed} = M_G \cdot \gamma_G + M_Q \cdot \gamma_Q = 40 \cdot 1.35 + 65 \cdot 1.50 = 151.5$ kNm

a) Table for the design of reinforced concrete rectangular cross-sections

Limit values for singly reinforced sections for B500 are (Table 1):

$\xi_{lim} = x/d = 0.530$, $\mu_{Rd,lim} = 0.334$ and $\xi_{lim} = = z/d = 0.780$

Dimensionless design bending moment - expression (43):

$$\mu_{Ed} = \frac{M_{Ed}}{b \cdot d^2 \cdot f_{cd}} = \frac{15150}{30 \cdot 61^2 \cdot 1.67} = 0.081 < \mu_{Rd,lim} = 0.334$$

Since $\mu_{Ed} < \mu_{Rd,lim}$ this is the case of singly reinforced section. From Table 2 we read out (the first greater or equal value of μ_{Ed}):

$\xi = 0.111$, $\xi = 0{,}954$ and $\mu_{Ed} = 0.086$

The required reinforcement area according to expression (46) is:

$$A_{s1} = \frac{M_{Ed}}{\zeta \cdot d \cdot f_{yd}} = \frac{15150}{0.954 \cdot 61 \cdot 43.47} = 5.99 \text{ cm}^2$$

b) Direct analytical procedure

Limiting bending moment for singly reinforced section is obtained via expression (66):

$$M_{Rd,lim} = \mu_{Rd,lim} \cdot b \cdot d^2 \cdot f_{cd} = 0.334 \cdot 30 \cdot 61^2 \cdot 1.667 =$$
$$= 62141 \text{ kNcm} = 621.41 \text{ kNm}$$

Since M_{Ed} = 151.50 kNm < $M_{Rd,lim}$ = 621.41 this is the case of singly reinforced section.
Depth of neutral axis x – expression (62):

$$x = \frac{61}{2 \cdot 0.416}\left[1 - \sqrt{1 - \frac{4 \cdot 0.416 \cdot 15150}{0.810 \cdot 30 \cdot 61^2 \cdot 1.667}}\right] = 6{,}42 \text{ cm}$$

The required reinforcement area is obtained using expression (65):

$$A_{s1} = \frac{\alpha_v(-3.5‰) \cdot b \cdot x \cdot f_{cd}}{f_{yd}} = \frac{0.810 \cdot 30 \cdot 6.42 \cdot 1.667}{43.478} = 5{,}97 \text{ cm}^2$$

The required area of the tension reinforcement determined by the direct analytical procedure (5.97 cm²) is numerically accurate and will always be smaller than or equal to the required area of the tension reinforcement obtained by the tables for the design of reinforced concrete rectangular sections (5.99 cm²), because in design tables, we look for the first greater or equal μ_{Ed} from the calculated μ_{Ed} to be on the safety side.

5.2. Example 2

This example is the same as example 1, with the difference that the design bending moment equals to M_{Ed} = 100, 200, 300, 400, 500 and 600 kNm, while the characteristic compressive strengths of concrete are f_{ck} = 40, 45, 50, 55, 60, 70, 80 and 90 MPa. Design was carried out according to the standard HRN EN 1992-1-1 [1] (first generation of Eurocode 2) and the standard FprEN 1992-1-1 [10] (second generation of Eurocode 2). A bilinear stress-strain design diagram with a horizontal upper branch without strain limit was used for the reinforcing steel. For the design according to the HRN EN 1992-1-1 standard, α_{cc} = 1,0 is assumed, while the other parameters of the stress-strain diagram for concrete are listed in Table 3.
Parameters for concrete according to standard FprEN 1992-1-1 [10] (second generation of Eurocode 2) are listed in Table 4.

Table 3. Parameters for concrete according to HRN EN 1992-1-1 [1] (first generation of Eurocodes)

Parameters \ Concrete	C40/50	C45/55	C50/60	C55/67	C60/75	C70/85	C80/95	C90/105
f_{ck} [MPa]	40	45	50	55	60	70	80	90
ε_{c2} [‰]	2.0	2.0	2.0	2.2	2.3	2.4	2.5	2.6
ε_{cu2} [‰]	3.5	3.5	3.5	3.1	2.9	2.7	2.6	2.6
n	2.0	2.0	2.0	1.75	1.6	1.45	1.4	1.4

Table 4. Parameters for concrete according to FprEN 1992-1-1 [10](second generation of Eurocodes)

Parameters \ Concrete	C40/50	C45/55	C50/60	C55/67	C60/75	C70/85	C80/95	C90/105
f_{ck} [MPa]	40	45	50	55	60	70	80	90
ε_{c2} [‰]	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
ε_{cu} [‰]	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5
k_{tc}	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
η_{cc}	1.000	0.961	0.928	0.899	0.874	0.830	0.794	0.763

Table 5. Design results

M_{Ed} [kNm]	$(A_{s1.2G} - A_{s1.1G}) / A_{s1.1G}$ [%]							
	f_{ck} [MPa]							
	40	**45**	**50**	**55**	**60**	**70**	**80**	**90**
100	0.000	0.065	0.112	0.110	0.106	0.102	0.096	0.105
200	0.000	0.136	0.234	0.230	0.220	0.211	0.199	0.217
300	0.000	0.215	0.369	0.361	0.345	0.329	0.309	0.335
400	0.000	0.305	0.519	0.506	0.482	0.457	0.427	0.462
500	0.000	0.406	0.687	0.667	0.633	0.596	0.555	0.598
600	0.000	0.523	0.877	0.847	0.800	0.748	0.693	0.745

A total of 48 designs according to the first generation of Eurocode 2 and 48 designs according to the second generation of Eurocode 2 are required. For each design, the required reinforcement area according to the first generation of Eurocode 2 ($A_{s1,1G}$) and the required reinforcement area according to the second generation of Eurocodes ($A_{s1,2G}$) are calculated. The difference in the design results is given as $(A_{s1,2G} - A_{s1,1G}) / A_{s1,1G}$. If we obtain that $(A_{s1,2G} - A_{s1,1G}) / A_{s1,1G}$ is greater than 0, then we are on the safety side when designing according to the second generation of Eurocode 2, compared to the first generation of Eurocode 2, i.e., we obtain a larger reinforcement area when designing according to the second generation of Eurocode 2 compared to the first generation of Eurocode 2. The results of the design are shown in Table 5.

It can be seen that the difference $(A_{s1,2G} - A_{s1,1G}) / A_{s1,1G}$ is always positive, which means that by design according to the second generation of Eurocode 2, we get a larger reinforcement area compared to design according to the first generation of Eurocode 2. The biggest difference is for M_{Ed} = 600 kNm and f_{ck} = 50 MPa and amounts to $(A_{s1,2G} - A_{s1,1G}) / A_{s1,1G}$ = 0.877 %.

5.3. Example 3

In this example, a comparison of the design according to the first generation [1] and the second generation [10] of Eurocode 2 and the Regulation on technical standards for concrete and reinforced concrete [13] (PBAB) was carried out using the corresponding design tables [5, 18]. For this reason, the cross-section from Example 5.1a) is additionally designed according to the first generation of Eurocode 2 [1] and the Regulation on technical standards for concrete and reinforced concrete [13] (PBAB).

a) Design according to first generation of Eurocodes [1]

According to the first generation of Eurocodes [1], all values are the same as in the second generation of Eurocode 2 [10] (Example 5.1a) with the exception of the expression for the design compressive strength of concrete [1], although the same design compressive strength of concrete is obtained by both generations of Eurocode 2:

$$f_{cd} = \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_C} = 1.0 \cdot \frac{25}{1.5} = 16.67 \text{ MPa}$$

Limiting values for singly reinforced sections for strength classes from C12/15 to C50/60 [5] are:

$\xi_{lim} = x/d = 0{,}45$, $\mu_{Rd,lim} = 0.296$ and $\zeta_{lim} = z/d = 0{,}813$

Dimensionless design bending momenta [5]:

$$\mu_{Ed} = \frac{M_{Ed}}{b \cdot d^2 \cdot f_{cd}} = \frac{15150}{30 \cdot 61^2 \cdot 1.67} = 0.081 < \mu_{Rd.lim} = 0.296$$

Since $\mu_{Ed} < \mu_{Rd,lim}$, this is the case of singly reinforced section.

From tables [5] we read out (the first greater or equal value of μ_{Ed}):

$\varepsilon_{c,ed2}$ = -2.6 ‰, ε_{s1} = 20 ‰, ξ = 0.115, ζ =0.955 and μ_{Ed} =0.082

The required reinforcement area is calculated according to expression [5]:

$$A_{s1} = \frac{M_{Ed}}{\zeta \cdot d \cdot f_{yd}} = \frac{15150}{0.955 \cdot 61 \cdot 43.47} = 5.98 \text{ cm}^2$$

b) Design according to the Regulation on technical standards for concrete and reinforced concrete [13] (PBAB)

In design according to PBAB we use the same symbols as in the first [1] and second [10] generation of Eurocode 2, whenever possible. According to PBAB, global safety factors are used for actions, while there are no safety factors for materials. The safety factor for permanent actions is γ_G = 1.6 while the safety factor for variable actions is γ_Q = 1.8.

Design bending moment according to PBAB:

$M_{Ed} = M_G \cdot \gamma_G + M_Q \cdot \gamma_Q = 40 \cdot 1.6 + 65 \cdot 1.8 = 181$ kNm

According to PBAB, concrete classes denoted as MB are used, which represent the characteristic concrete compressive strength (10 % fractile) obtained on cubes with side length of 20 cm. According to both generations of Eurocode 2, the characteristic concrete compressive strength (5 % fractile) can also be obtained (except on cylinders with diameter of 15 cm and height of 30 cm) on cubes with side length of 15 cm.

According to the Regulation on technical standards for concrete and reinforced concrete [13] (PBAB) (Table 1), the ratio of the concrete compressive strength of a cube with side length of 20 cm to that of a cube with side length of 15 cm is 0,95. The characteristic concrete compressive strength of a cube with side length of 15 cm is 30 MPa for concrete C25/30, according to both generations of Eurocode 2.

Therefore, the concrete class MB according to PBAB (characteristic concrete compressive strength of a cube with side length of 20 cm) is 95 % value of the characteristic concrete compressive strength of a cube with side length of 15 cm for concrete C25/30:

$$MB = 0.95 \cdot 30 = 28.50 \text{ MPa}$$

The concrete class MB 28,50 corresponds to the design compressive strength f_B = 19.52 MPa (obtained according to [13] by linear interpolation from table 15).

Dimensionless design bending moment according to [18]:

$$\mu_{Ed} = \frac{M_{Ed}}{b \cdot d^2 \cdot f_B} = \frac{18100}{30 \cdot 61^2 \cdot 1.952} = 0.083$$

From the tables for design of rectangular cross-sections according to PBAB [18], the first greater or equal μ_{Ed} is selected:

$\varepsilon_{c,ed2}$ = -1.7 ‰, ε_{s1} = 10 ‰, ξ = 0.145, ζ =0.947 and μ_{Ed} = 0.084

According to PBAB, the limit strain in concrete is -3.5 ‰ while the limit strain in reinforcement is 10 ‰.

The required reinforcement area is calculated according to [18] (according to PBAB, the denominator is not the design yield strength of the reinforcement but the characteristic yield strength):

$$A_{s1} = \frac{M_{Ed}}{\zeta \cdot d \cdot f_{yk}} = \frac{18100}{0.947 \cdot 61 \cdot 50} = 6.27 \text{ cm}^2$$

c) Comparison of results

According to the first generation of Eurocode 2 (Example 3a), the area of the tensile reinforcement is 5.98 cm², whereas according to the second generation of Eurocode 2 (Example 1a), the area of the tensile reinforcement is 5.99 cm². This difference is due to the use of different design tables and the first reading greater than or equal to μ_{Ed} to be on the safe side. If the same tables were used, the same results would be obtained for both generations of Eurocode 2.

Design according to PBAB results in a slightly larger reinforcement area (6.27 cm²) than for both generations of Eurocode 2, but this is due to the use of different safety factors.

6. Conclusion

Tables for the design of reinforced concrete rectangular cross-sections are a basic tool for the design of rectangular sections subjected to bending. With the help of such tables, reinforced concrete rectangular cross-sections can be designed quickly and easily. This is particularly useful with the widespread use of computer programs for structural design with the possibility of automatic design of members at the time of the introduction of the second generation of structural Eurocodes. At least in the phase of adaptation to the new standards, structural engineers will need a reliable tool for checking the results obtained with specialized computer programs.

In this paper a new procedure for creation of tables for the design of reinforced concrete rectangular cross-sections was proposed and implemented in a way to vary the mechanical reinforcement ratio ω_1, since the required reinforcement area depends linearly on the mechanical reinforcement ratio. The advantage of this procedure is that the same maximum absolute error is obtained for the required reinforcement area, which corresponds to the varying step of the mechanical reinforcement ratio (0.010). The design tables for reinforced concrete rectangular cross-sections are made according to the standard FprEN 1992-1-1 (second generation of Eurocode 2). In addition to the tables, diagrams have also been created to simplify the design of members subjected to bending. The diagrams can be used to calculate the required reinforcement area and to determine the moment of resistance of the section.

A direct analytical procedure has been proposed and implemented, that can be used for the design of reinforced concrete rectangular cross-sections without the need to use tables, that is numerically accurate and in which the problem is reduced to the solution of a quadratic equation. It is shown that in the case of a reinforced concrete rectangular cross-section, with the concrete stress-strain diagram in the form of parabola-rectangle, the design can be carried out by applying the direct analytical procedure.

The paper compares the design of cross-sections with concrete stress-strain design diagrams according to both the first and the second generation of Eurocode 2. It was shown that the differences in the required reinforcement area according to these two generations of standards are negligible, which proves that the simplifications of the concrete stress-strain diagram in the second generation are on the safety side and rational.

The authors of the paper believe that a wider application of the stress-strain design diagram for concrete in the form of a rectangle (uniform stress distribution) should be considered as it allows the application of a direct analytical procedure for rectangular and T-sections. The use of the concrete stress-

strain design diagram in the form of a rectangular stress distribution is permitted by the FprEN 1992-1-1 standard and simplifies the design of reinforced concrete sections.
In addition, the authors of the paper propose to always limit the strain in the compression zone in concrete to -3.5 ‰ (rotation around point A) for design of reinforced concrete section, as this allows for a simpler design procedure that can also be used when creating the interaction diagrams. Although the standard FprEN 1992-1-1 does not define limit values for pure bending, a proposal for limit values for pure bending has also been made.

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